Computer-aided Design and Manufacturing

Adrian Bowyer

A.Bowyer@bath.ac.uk

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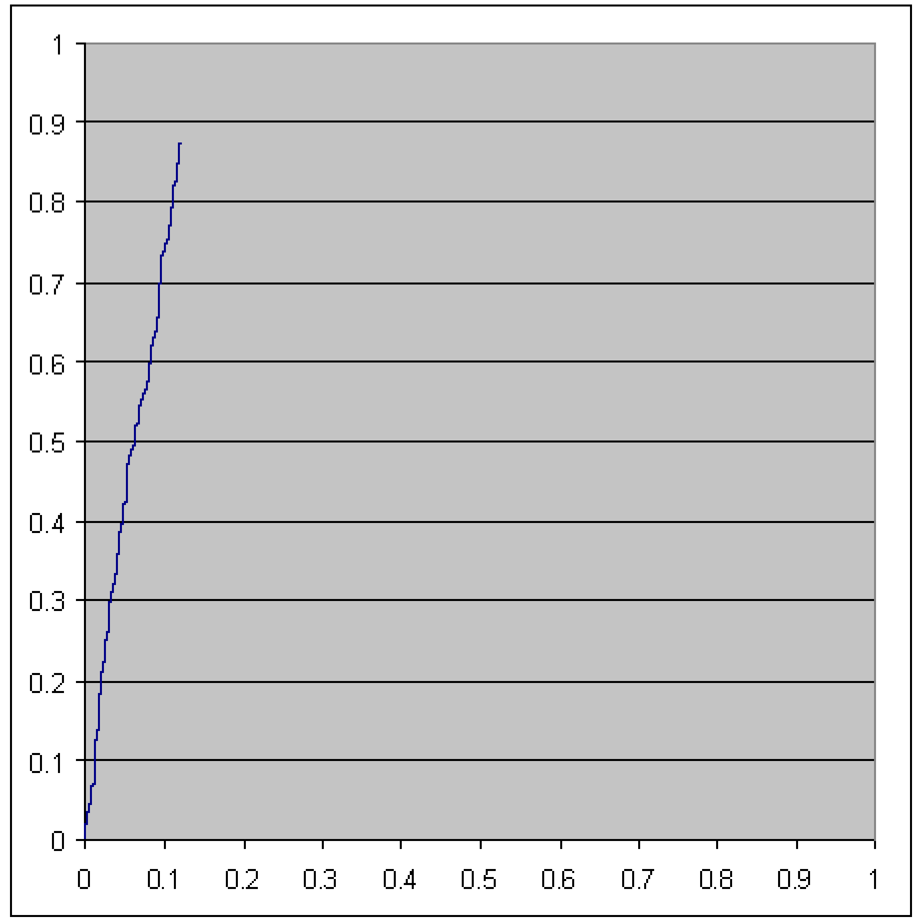
The Polya Urn – seemingly irrelevant...

1. Place one red and one white ball in the urn
2. Take a ball out at random
3. Replace it
4. Add another ball of the same colour
5. Go to 2...

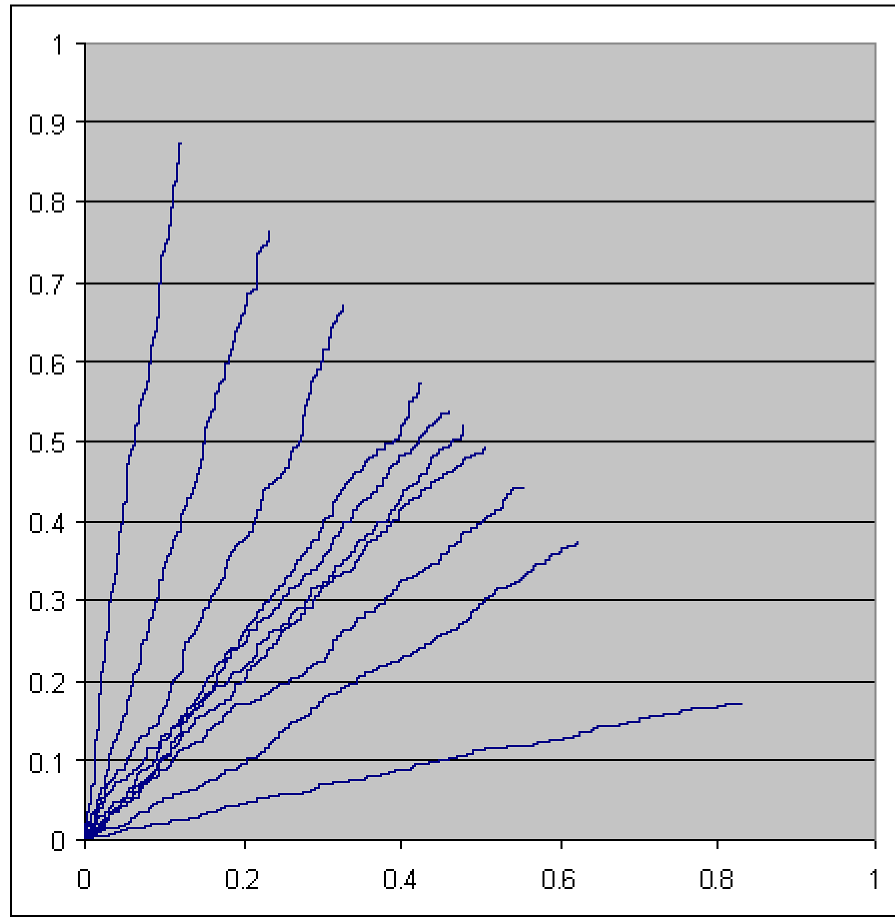
When there are, say, 1000 balls in the urn, what is the expected ratio of reds to whites?

The Polya Urn



A *Martingale* stochastic process

The Polya Urn



The *gradient* is random

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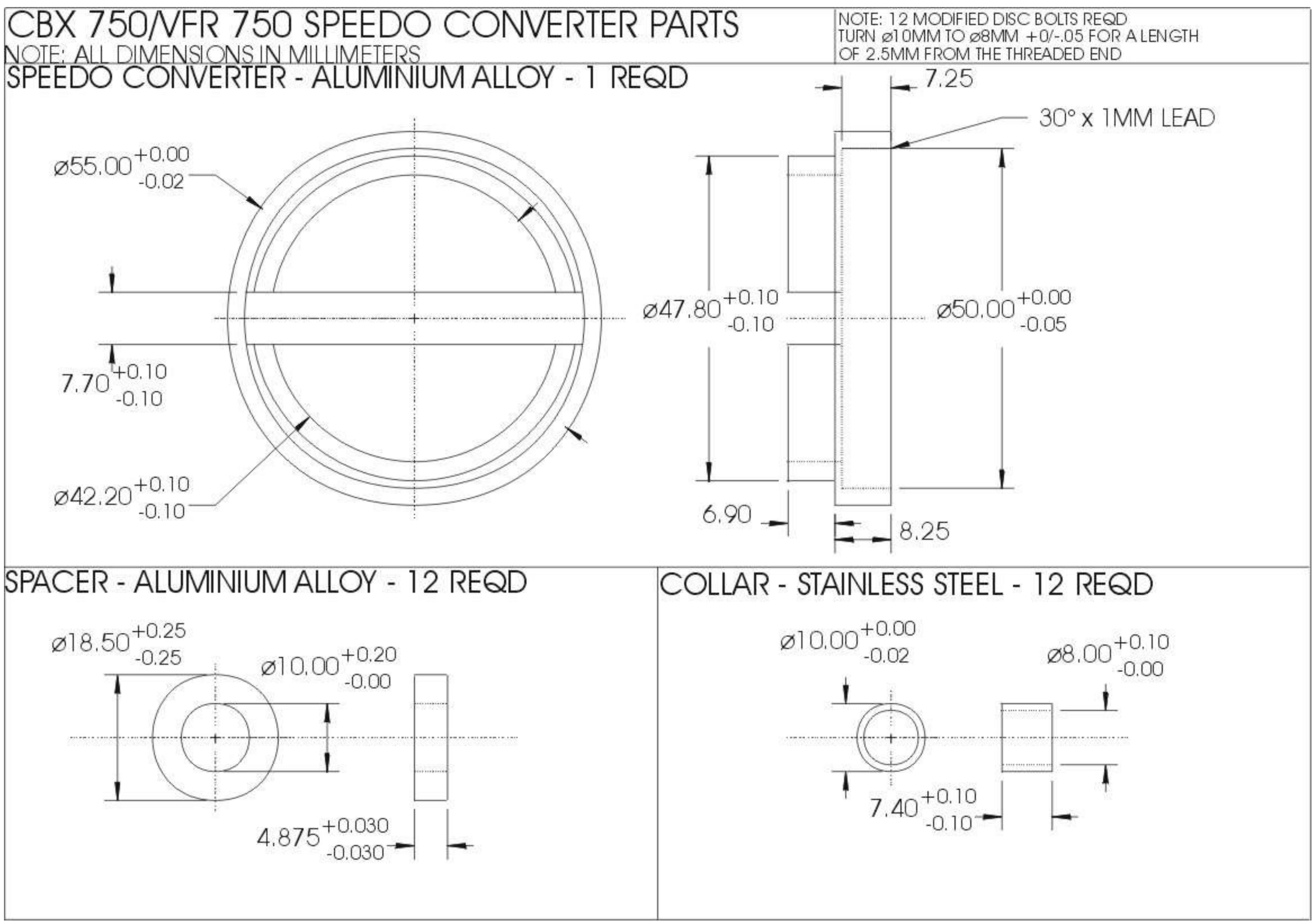
As soon as computer graphics became possible (late 1960s; Evans & Sutherland), people started to write programs to do engineering drawing:



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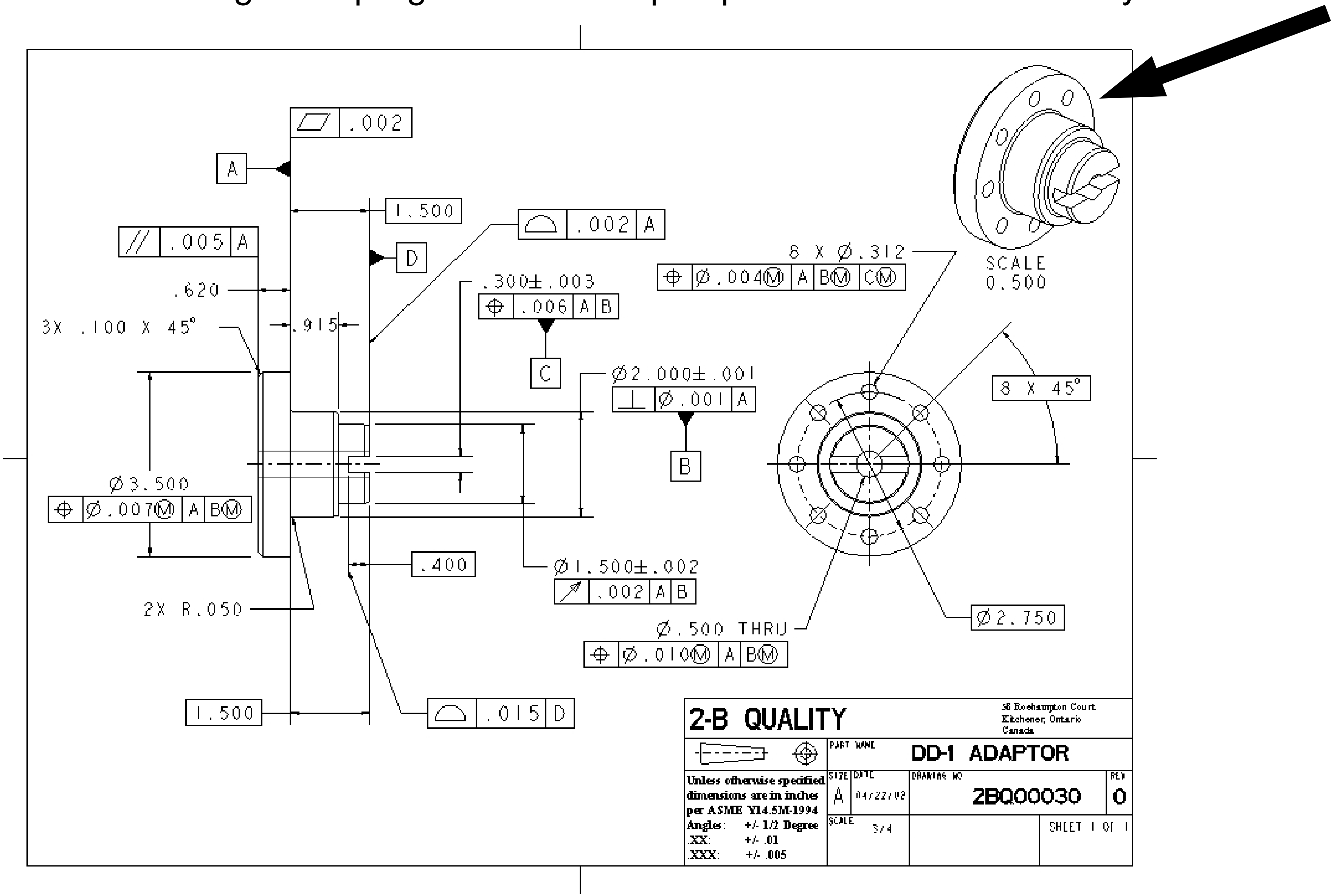
Great! Let's get the program to do the perspective view automatically.



Oh dear! It's impossible...

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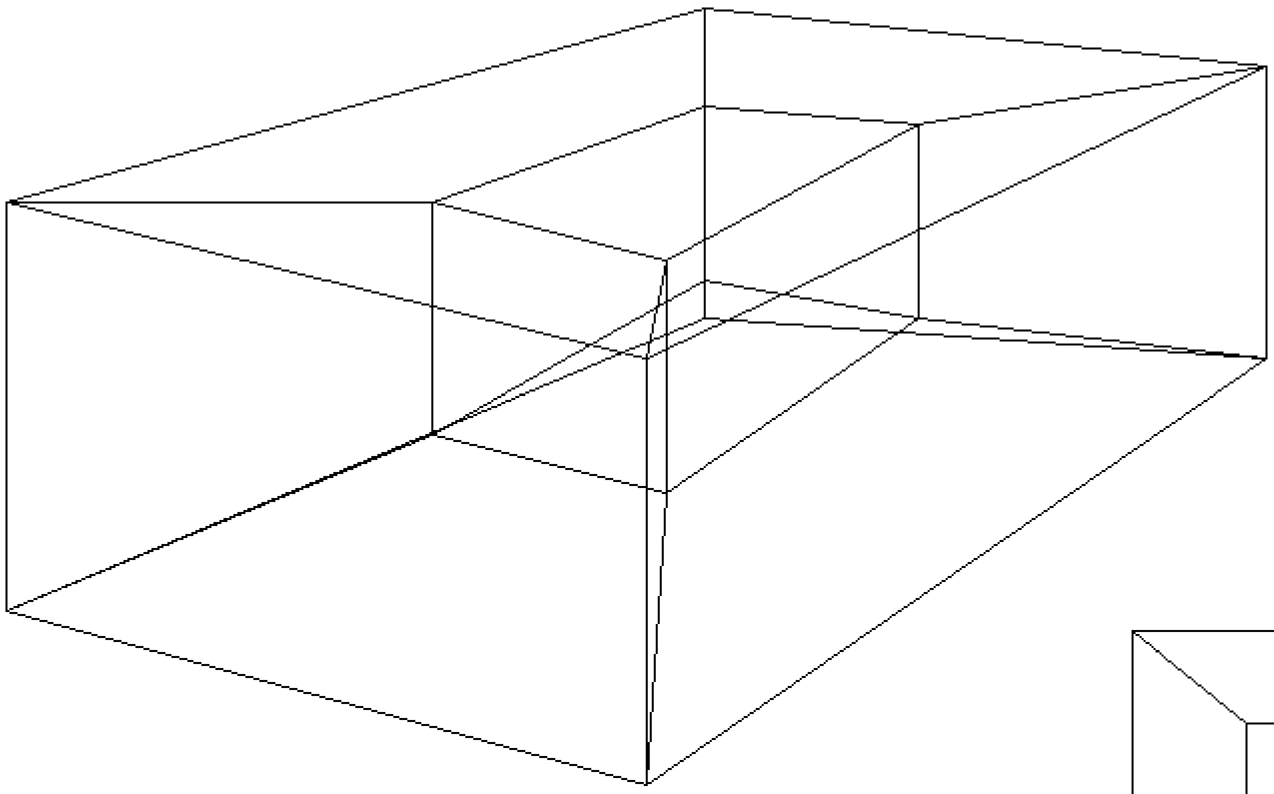
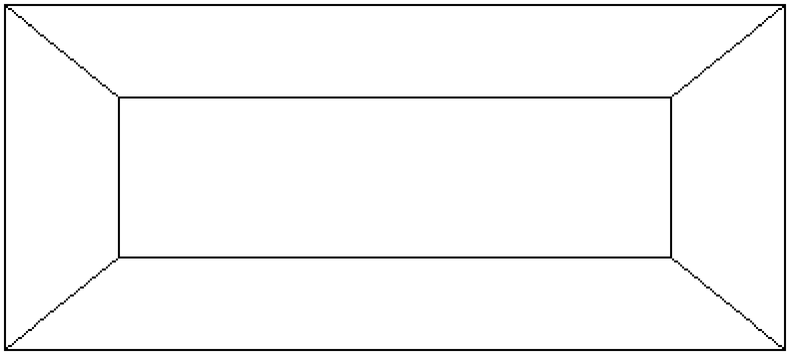
Why is it impossible?





We need an ***unambiguous*** 3D representation.

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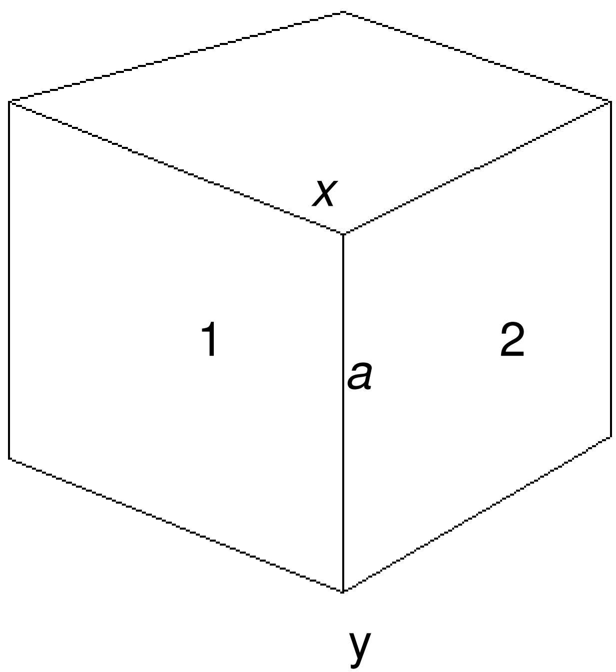
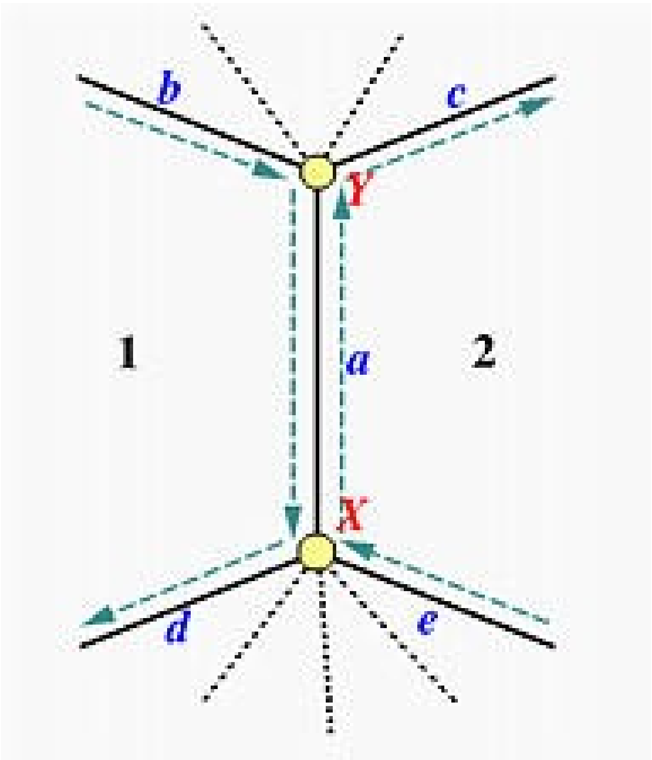
In fact, we need an unambiguous 3D representation for *all* automated downstream processes such as:

1. Volume
2. Mass
3. Surface area
4. Moments of inertia
5. Strength
6. Flexibility
7. Heat and fluid flow
8. Fields, currents and fluxes
9. Mechanical integrity and fit
10. Manufacture, and
11. Pictures

Computer Aided Design

Idea Number One – **The Boundary Representation** (Ian Braid, Bruce Baumgart *et al.*)





Extended Euler-Poincarré formula:

Faces + Vertices – Edges – Rings = 2(Shells - Holes)

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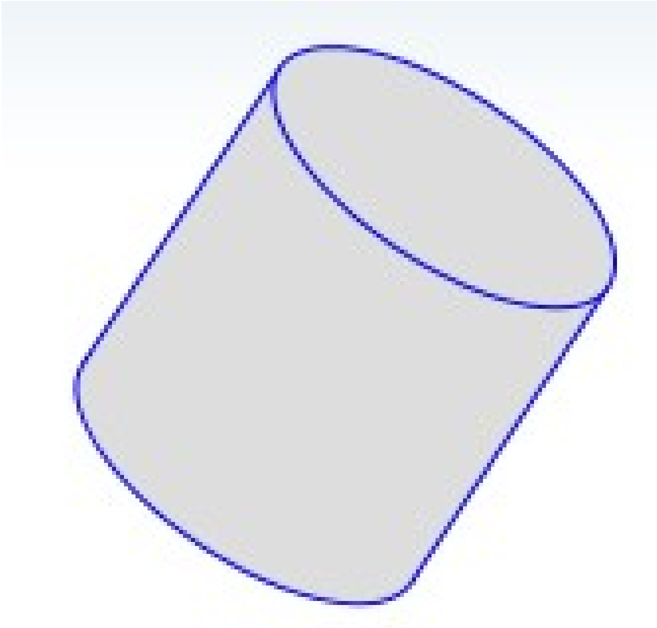
Idea Number One – **The Boundary Representation** (Ian Braid, Bruce Baumgart *et al.*)

The extended Euler-Poincarré formula allows us to test the *topology* for solidity, but...



Faces = 3
Vertices = 0
Edges = 2

It can't do non-polyhedral shapes directly.

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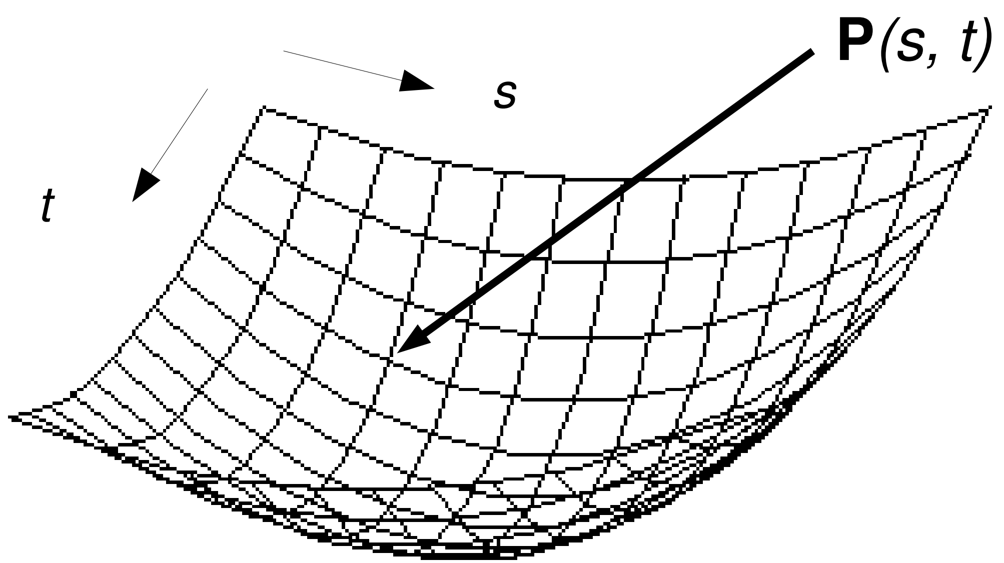
Idea Number One – **The Boundary Representation** (Ian Braid, Bruce Baumgart *et al.*)

Curved surfaces: **Bi-parametric patches**



$$\mathbf{P}(s, t) = (x(s, t), y(s, t), z(s, t))$$

The most common patch is the Non-uniform Rational B-spline: NURBS.

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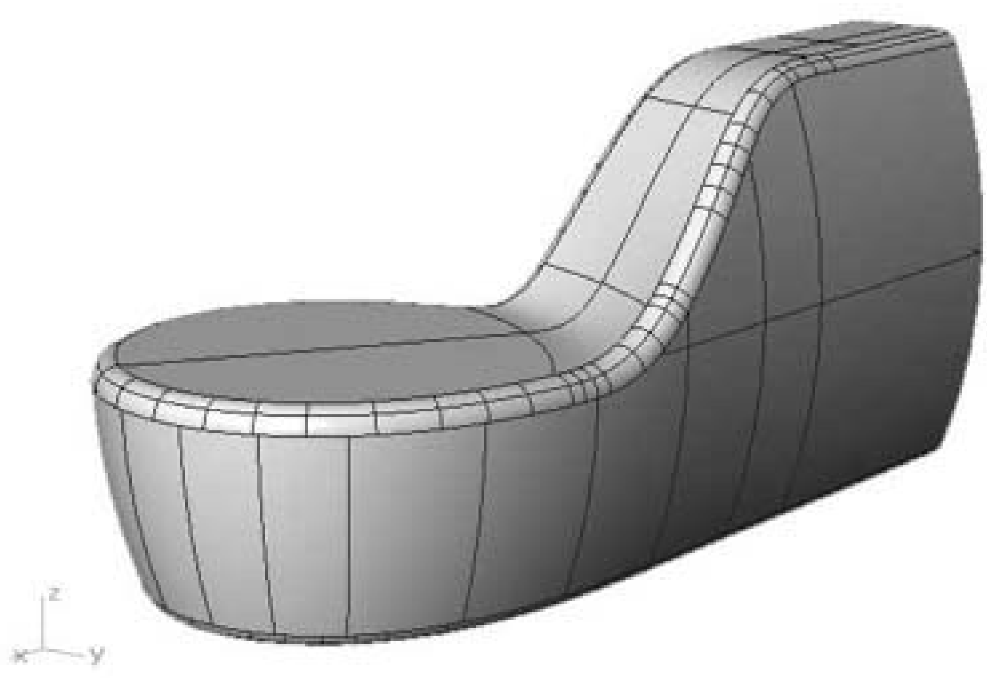
Idea Number One – **The Boundary Representation** (Ian Braid, Bruce Baumgart *et al.*)

Curved surfaces: **Bi-parametric patches**



We can now stitch these together to make objects as a patchwork quilt.

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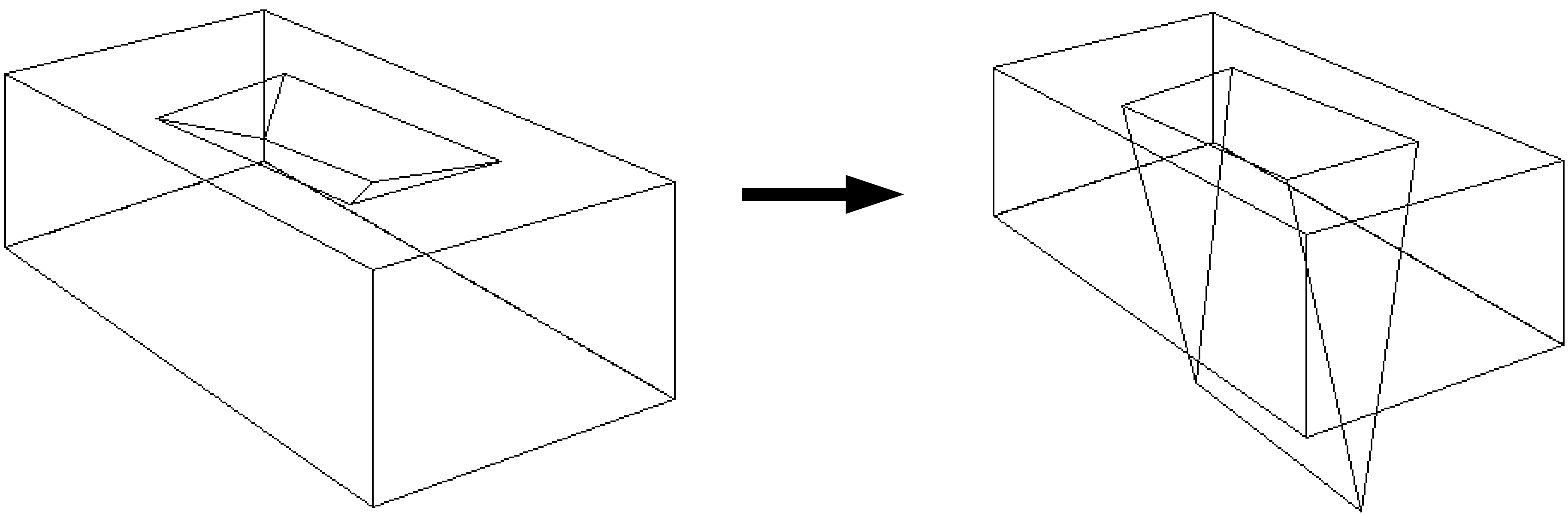
Idea Number One – **The Boundary Representation** (Ian Braid, Bruce Baumgart *et al.*)



But, if we change the geometry, the topology can become nonsense.

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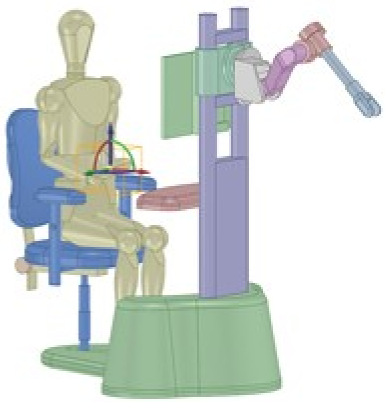
Idea Number One – **The Boundary Representation** (Ian Braid, Bruce Baumgart *et al.*)

These problems have been solved (largely...).

Some commercial B-Rep Geometric Modellers:



ACIS (Spatial Corp.)

Parasolid (Siemens PLM Software)

Open CASCADE (Open CASCADE S.A.S.)

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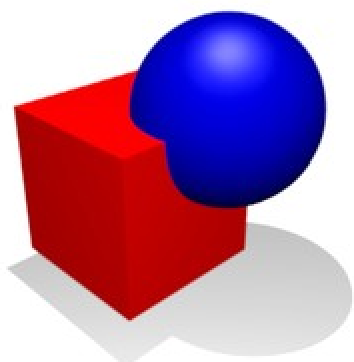
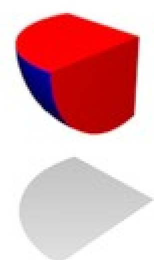
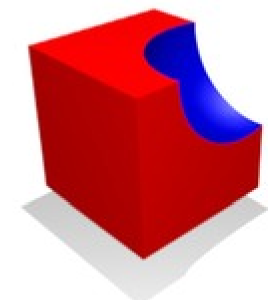
Idea Number Two – **The Constructive Solid Geometry Representation** (Ari Requicha, John Woodwark *et al.*)



Union



Intersection



Difference

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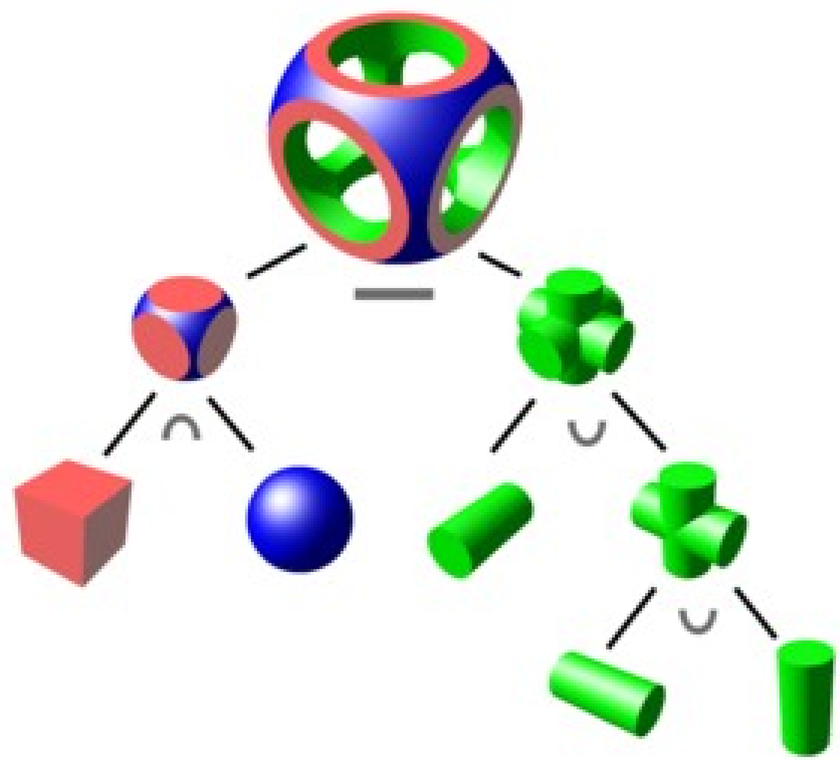
Idea Number Two – **The Constructive Solid Geometry Representation** (Ari Requicha, John Woodwark *et al.*)



The design is represented as an operator tree with geometric primitives at the leaves.

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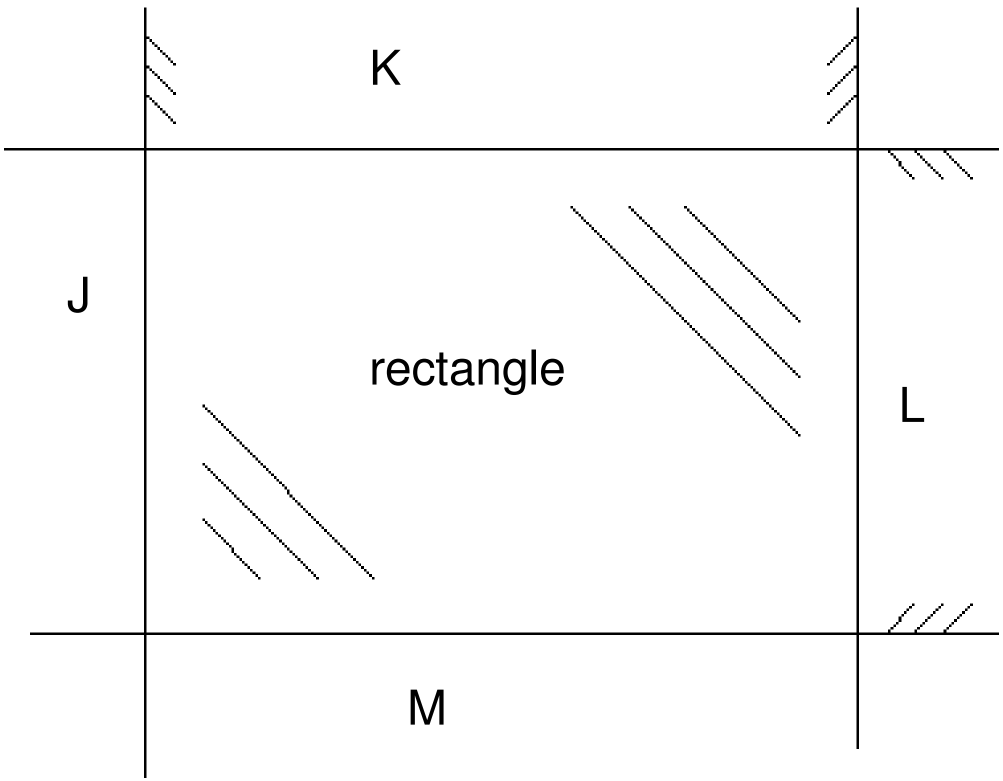
Idea Number Two – **The Constructive Solid Geometry Representation** (Ari Requicha, John Woodwark *et al.*)



J: $Ax + By + C <= 0$ *etc.*

(Convention: negative is solid)

Rectangle: $J \wedge K \wedge L \wedge M$

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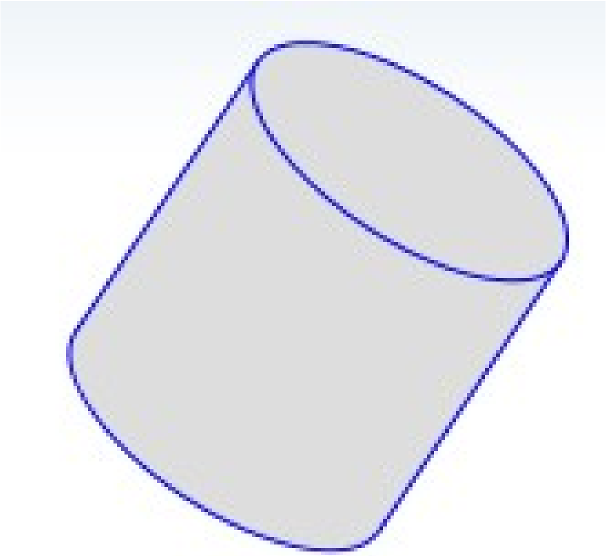
Idea Number Two – **The Constructive Solid Geometry Representation** (Ari Requicha, John Woodwark *et al.*)

Infinite cylinder, I: $x^2 + y^2 - r^2 <= 0$
Infinite planar half-space, P : $Ax + By + Cz + D <= 0$

Cylinder with ends: $I \wedge P_1 \wedge P_2$



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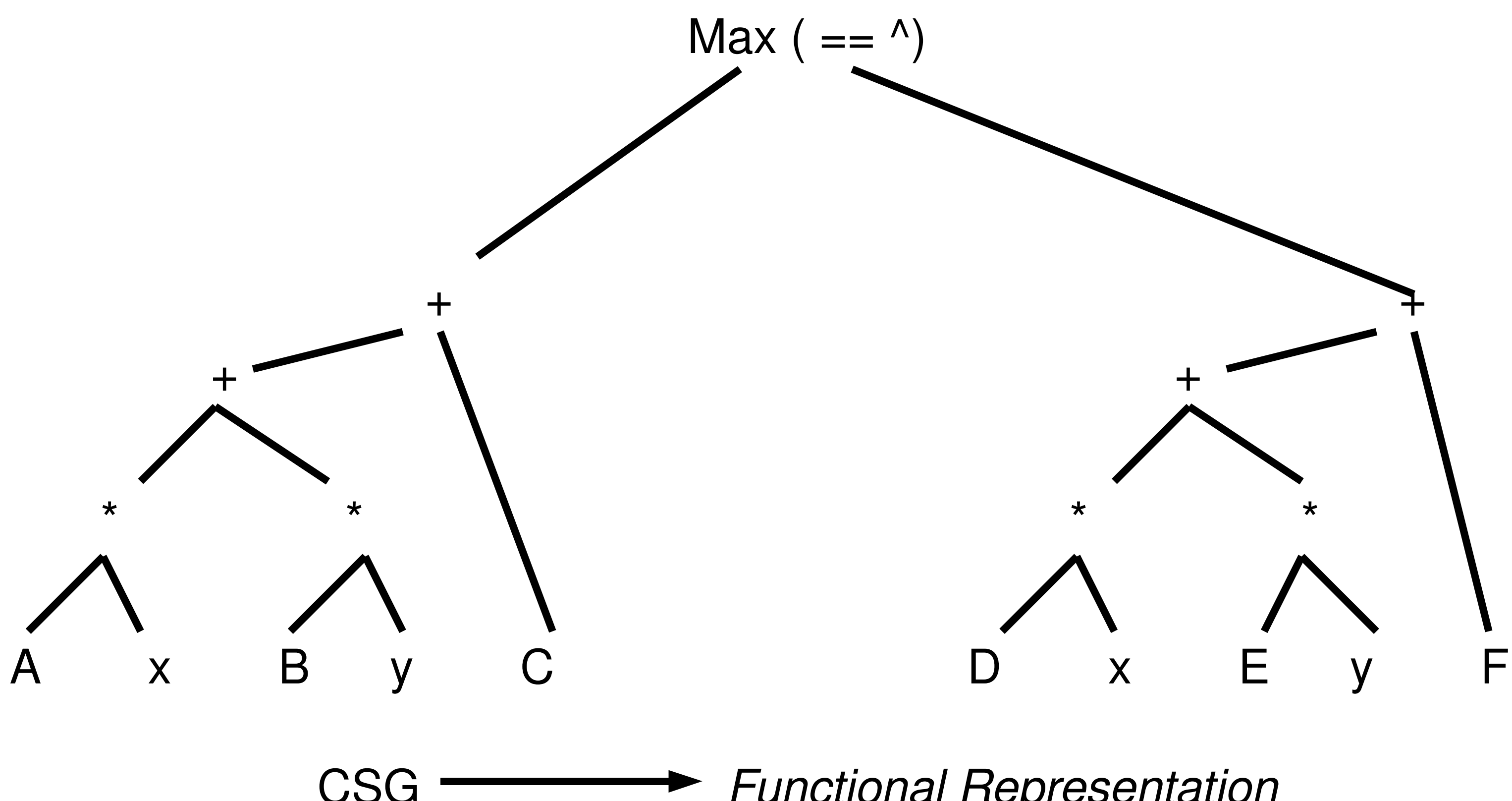
Idea Number Two – **The Constructive Solid Geometry Representation** (Ari Requicha, John Woodwark *et al.*)

And look! We can mix all the algebra and set-theory up in the tree.

Remembering the convention that negative is solid:



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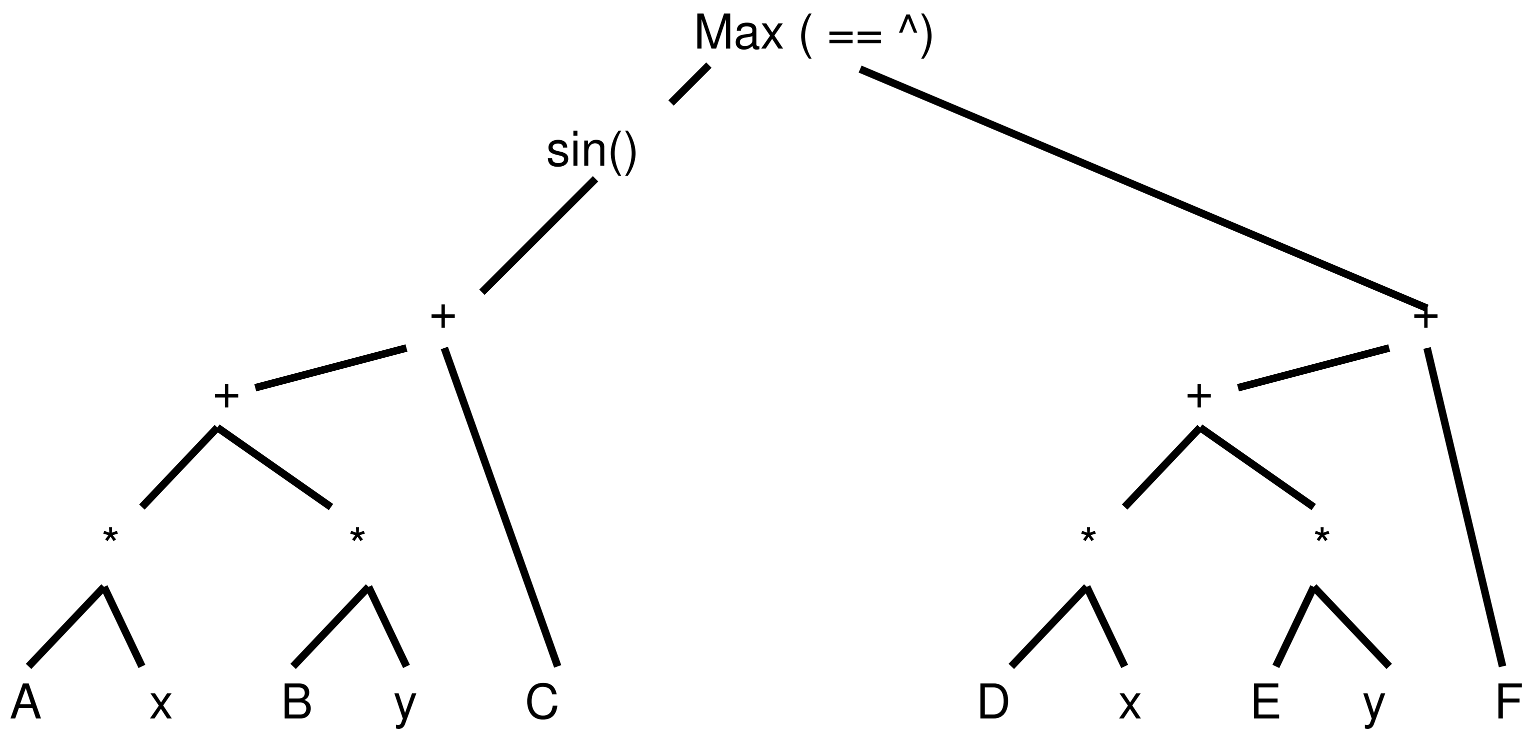
Idea Number Two - **The Functional Representation**

We can put in any operations and functions that we like: sin(...), $(...)^3$, ln(...) and so on.



This gives us all the bendy surfaces that parametric patches gave B-Rep, and more.

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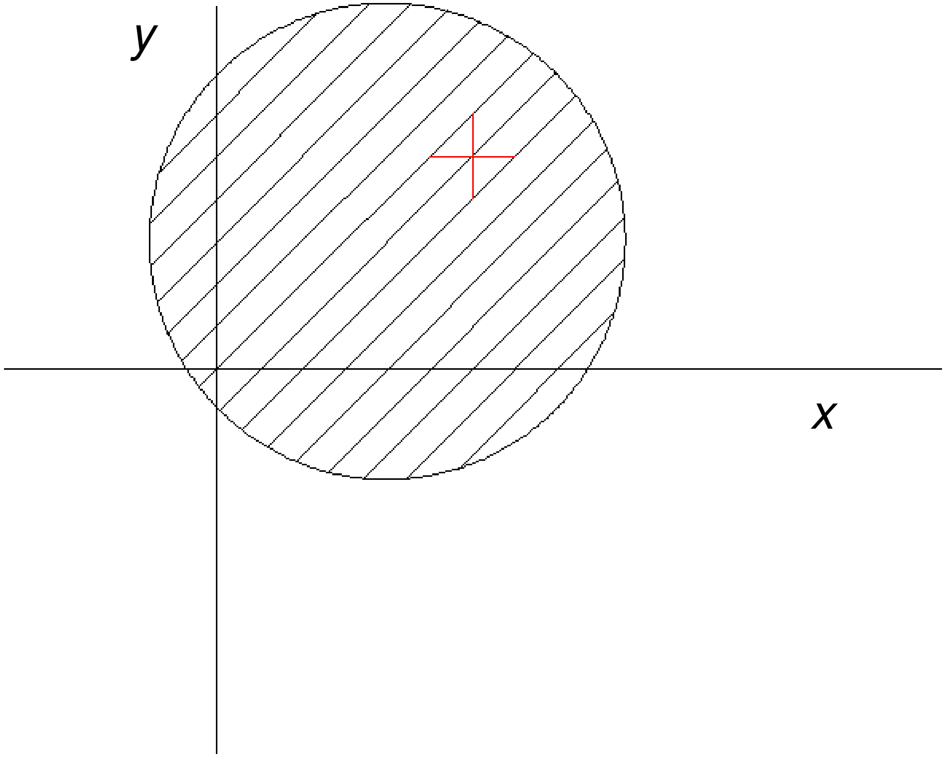
Idea Number Two - **The Functional Representation**

We will (almost) always have a valid unambiguous solid, but...

We don't know where it is or what it is shaped like.

We have to **evaluate** it.



Disc: $(x - a)^2 + (y - b)^2 - r^2 <= 0$

Point: (x_1, y_1)

Evaluate: $(x_1 - a)^2 + (y_1 - b)^2 - r^2$

→ **-** : inside

→ 0 : on surface (caution: f.p. arithmetic)

→ **+** : outside

A **point membership test**.

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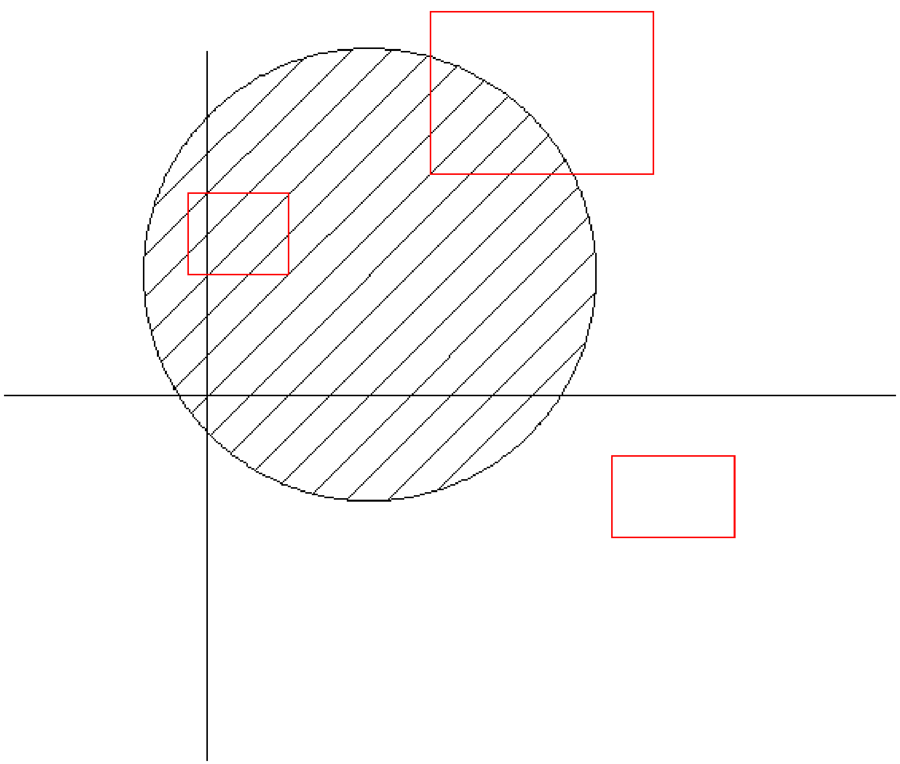
Idea Number Two - **The Functional Representation**

Evaluating rectangles rather than points.



Disc: $(x - a)^2 + (y - b)^2 - r^2 <= 0$

Rectangle: $([x_{low,}, x_{high}], [y_{low,}, y_{high}]) = (\boldsymbol{X}_i, \boldsymbol{Y}_i)$

Evaluate: $(\boldsymbol{X}_i - a)^2 + (\boldsymbol{Y}_i - b)^2 - r^2$

→ [-, -]: definitely inside
→ [-, +]: ***May*** straddle surface
→ [+, +]: definitely outside

An **interval membership test**.

It is **conservative**.

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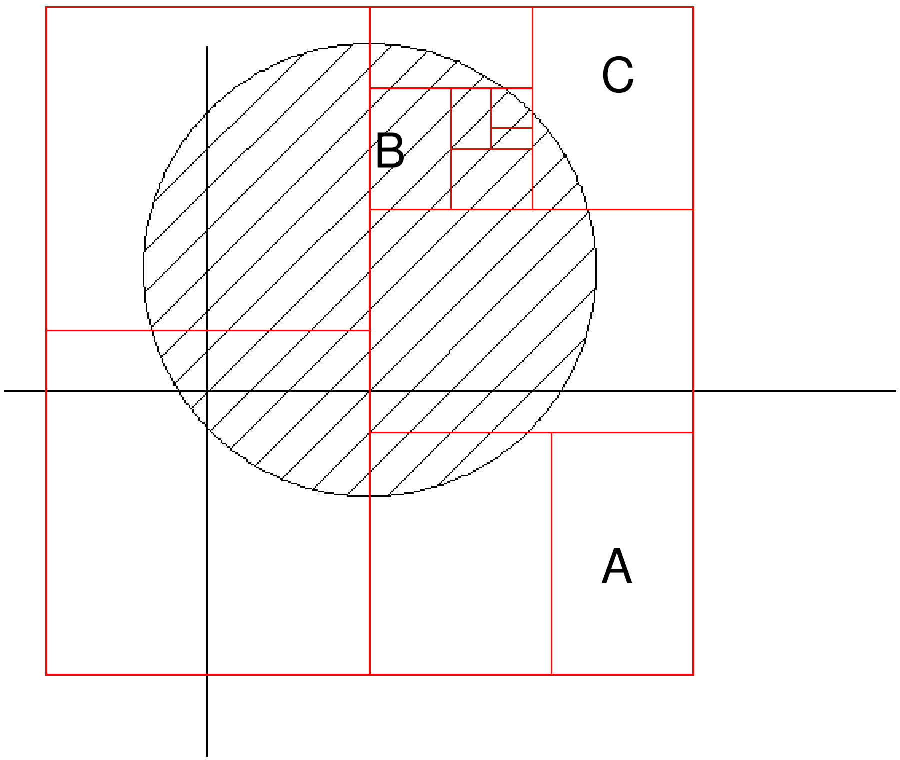
Idea Number Two - **The Functional Representation**

Evaluating the entire shape using intervals.



Recursive spatial division (e.g. a quad tree, or – as here – a binary tree).

Rectangle A is entirely void (= *null set*).

Rectangle B is entirely solid (= *universal set*).

Rectangle C contains surface.

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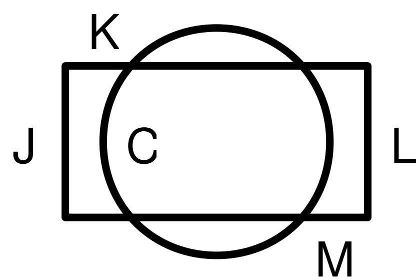
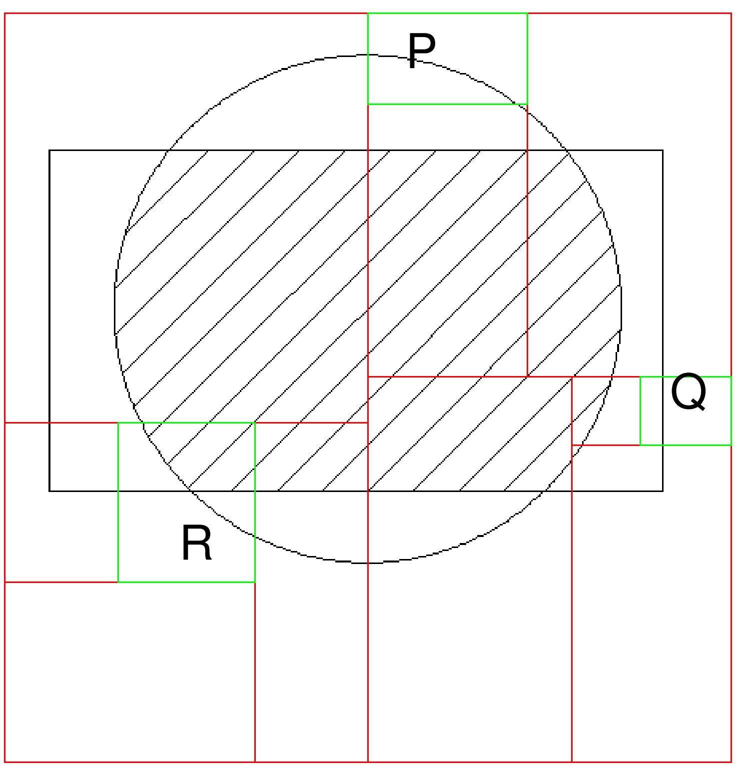
Idea Number Two - **The Functional Representation**

Pruning the tree.





The hatched area is

C ^ J ^ K ^ L ^ M

In Q: C gives ***null***

→ ***null*** ^ J ^ K ^ L ^ M
→ whole thing is ***null*** in Q

In P: K gives ***null***

→ whole thing is ***null*** in P

In R: J, K, and L give ***universal***

→ C ^ **true** ^ M
→ we ***MAY*** have surface in R

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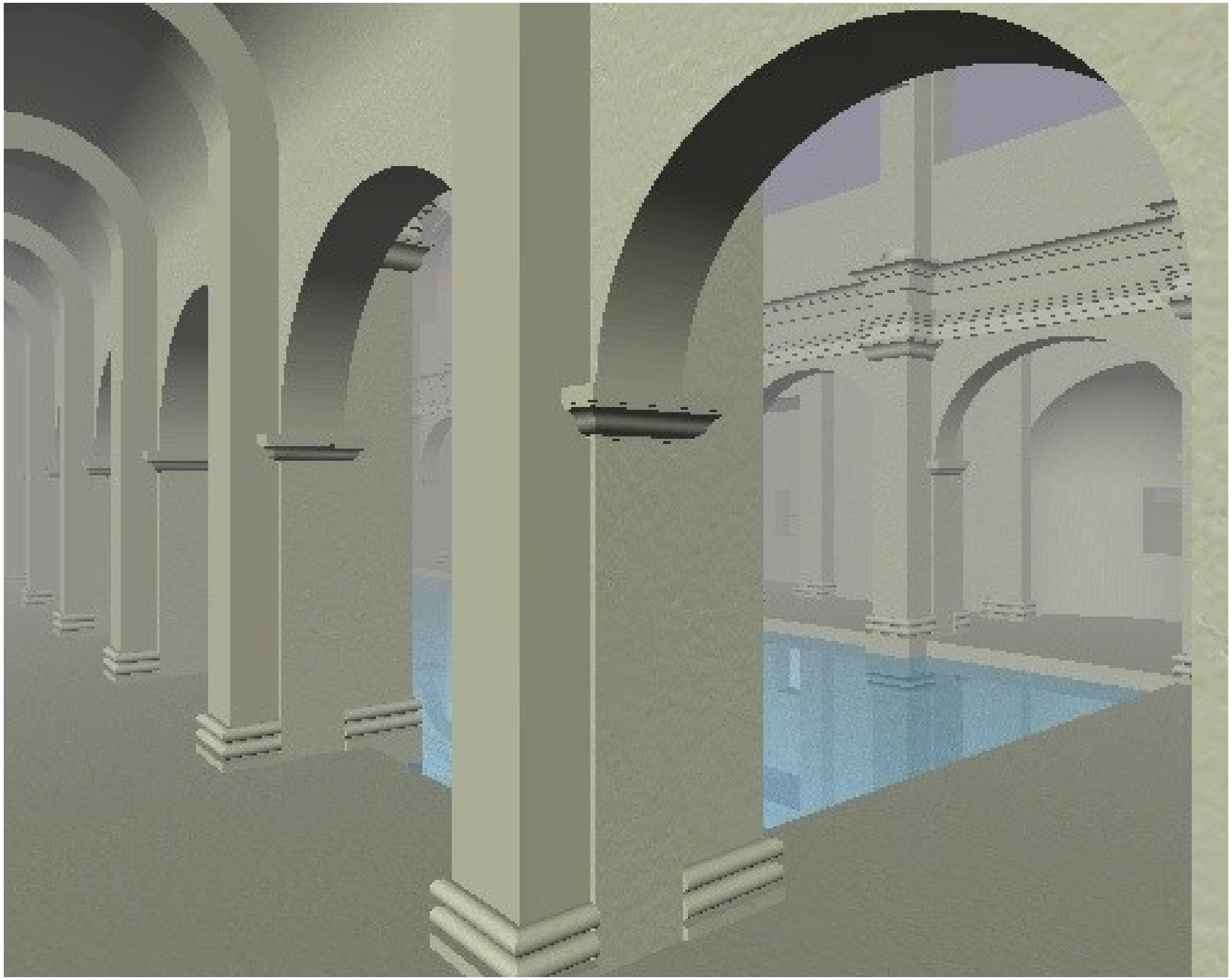
Idea Number Two - **The Functional Representation**

Pruning the tree.



The Great Bath

Aquæ Sulis

Roman Britain

Computer Aided Design

Boundary Representation vs. Functional Representation

B-Rep	**F-Rep**
Bad: Calculating volume (hence mass) is hard Numerical accuracy problems The data structures use lots of memory Input can be messy without lots of extra software The model is evaluated *Good:* Easy to triangulate the surface Making pictures is easy Calculating surface area is easy B-rep primitives are local The model is evaluated	*Bad:* Making pictures is hard Calculating surface area is hard F-rep primitives are not local The model is unevaluated Hard to triangulate the surface *Good:* Calculating volume (hence mass) is easy Easy(ish) to make numerically robust Input is pretty straightforward The model is unevaluated The data structures are compact

Computer Aided Design

Boundary Representation vs. Functional Representation

Right! We can now take our prefered representation and go away and design Lamborghinis, wave guides and art galleries!

Errr. No. We need a user-interface. And that typically takes much more code and many more person-hours than the geometric modeller.

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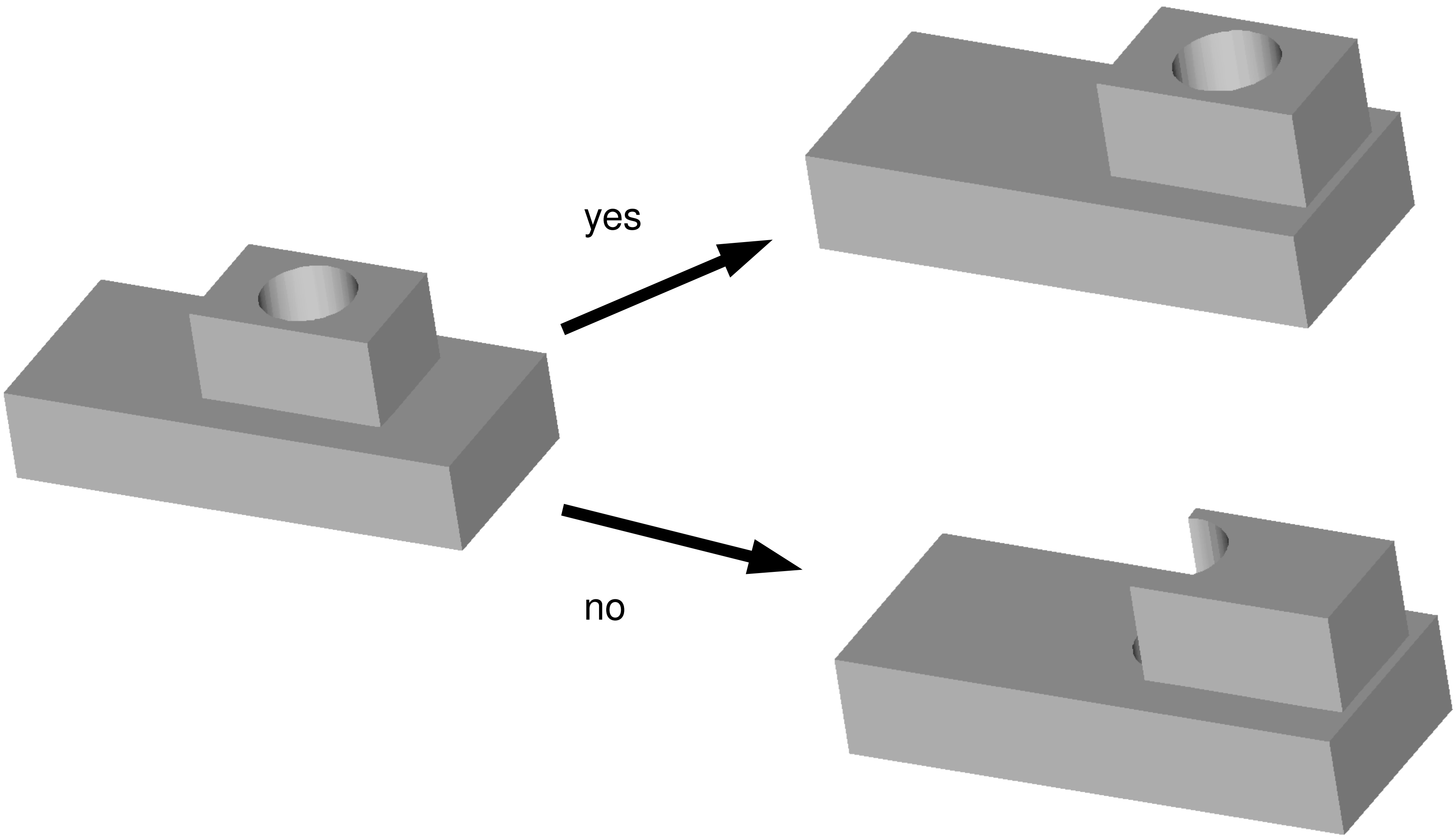
One aspect of the User Interface – **Geometric Constraints**



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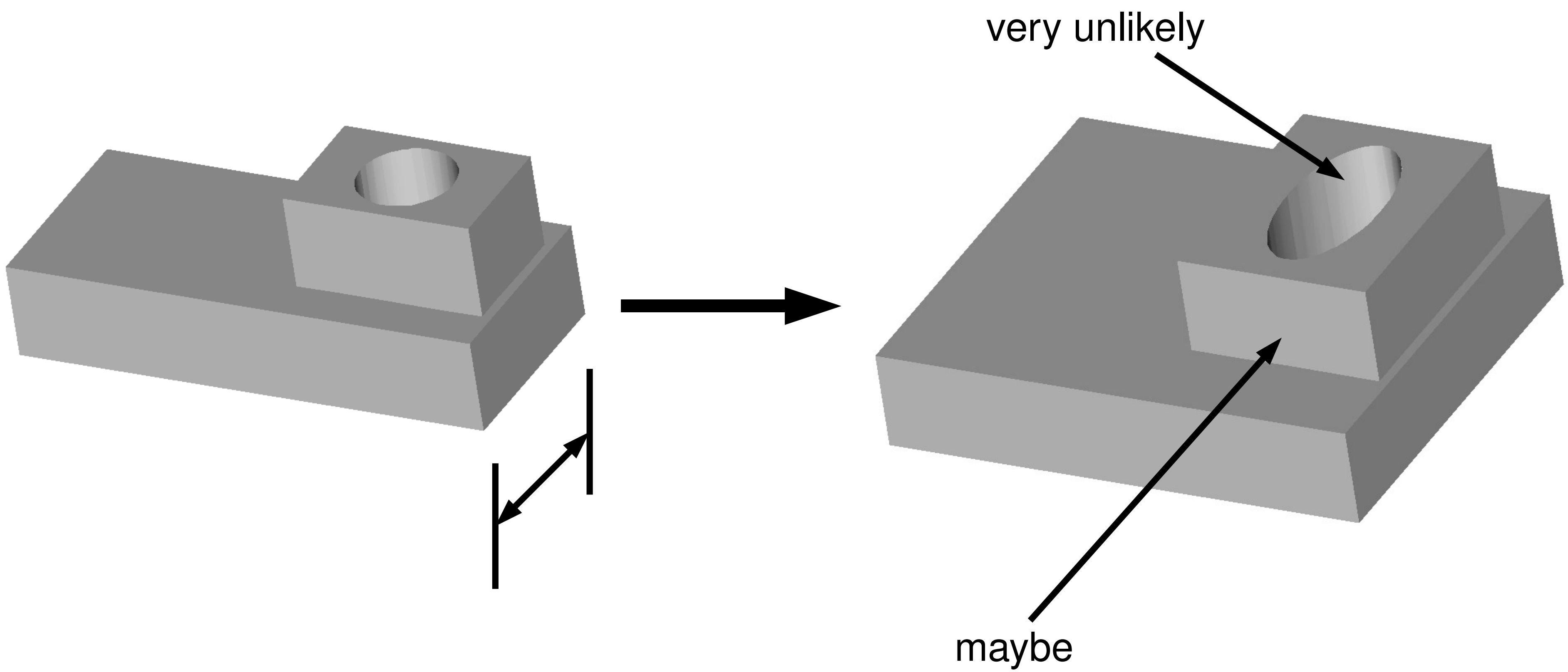
One aspect of the User Interface – **Geometric Constraints**



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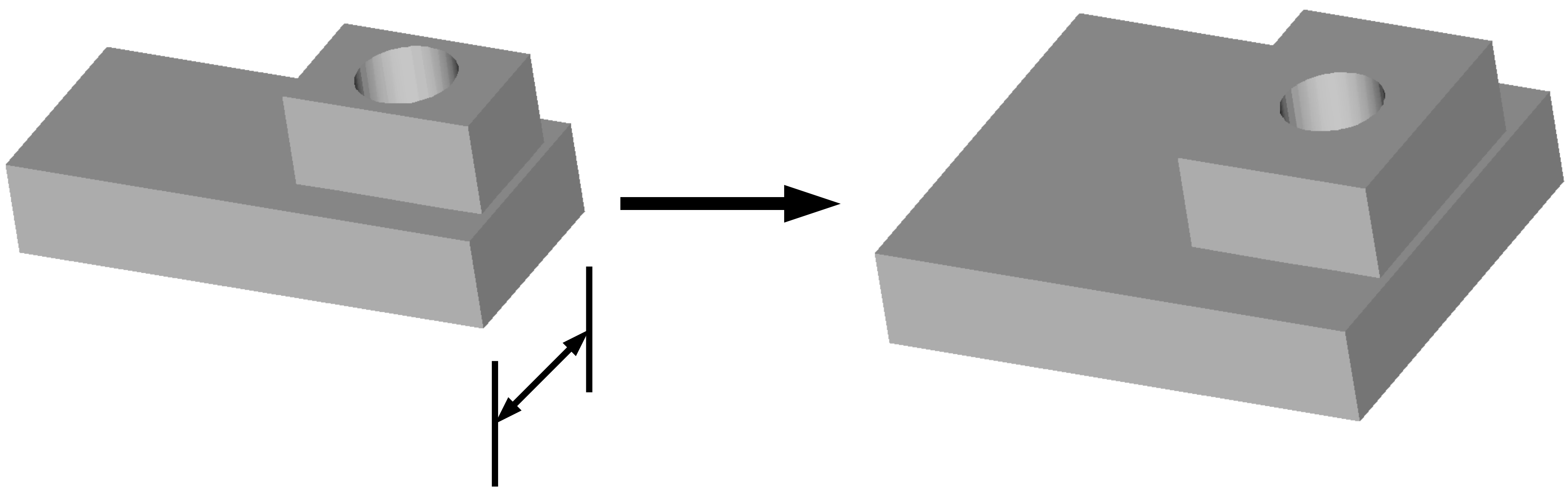
One aspect of the User Interface – **Geometric Constraints**

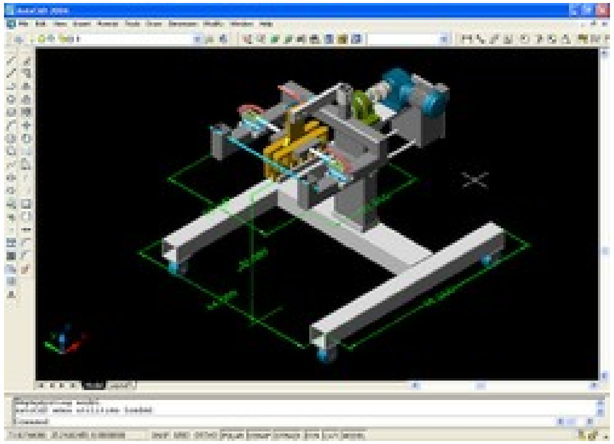


We end up with a large hierarchical non-linear geometric system to solve **on-the-fly as the user is designing**.

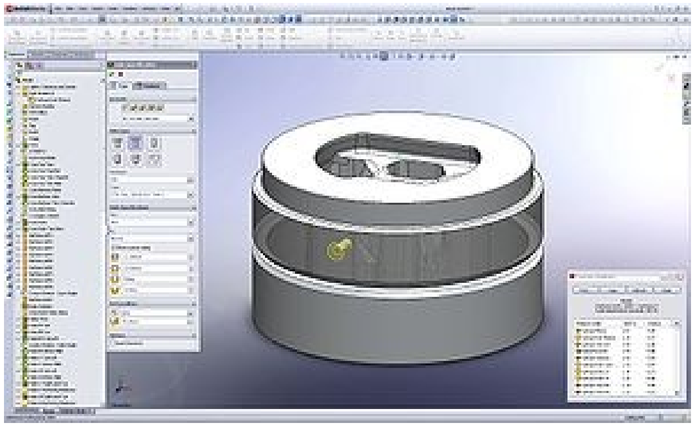
Computer Aided Design

Some commercial systems

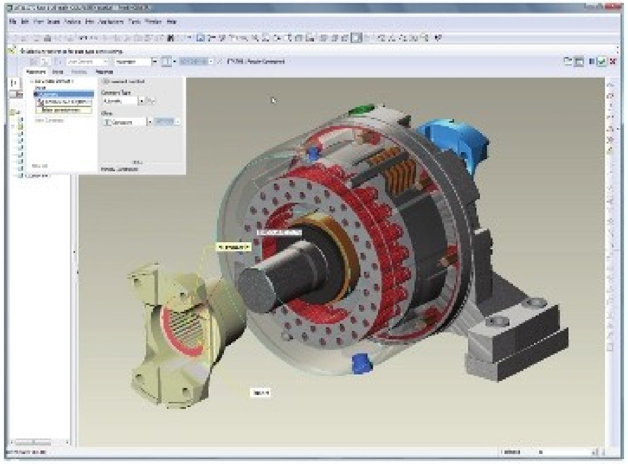
AutoCAD (Autodesk Inc.)



SolidWorks (Dassault)



Pro-Engineer (Parametric Technology)



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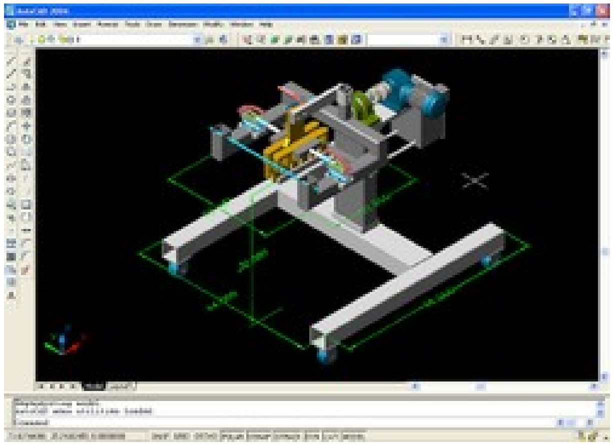
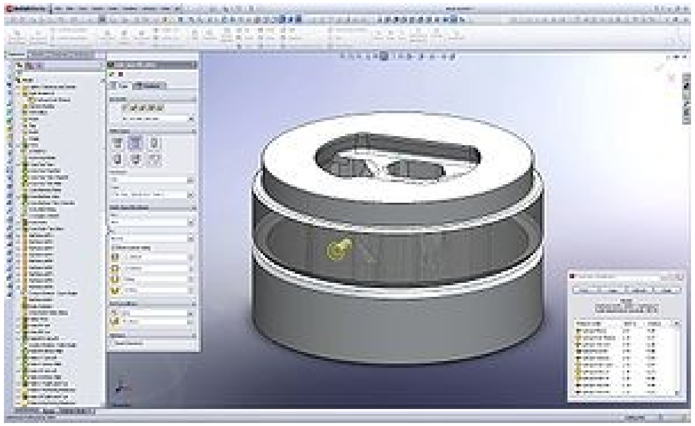
All commercial systems are B-Rep.



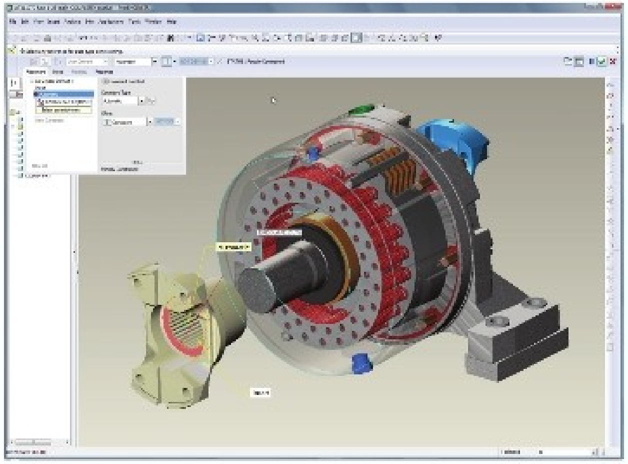
They grew out of 2D drafting

B-Rep is quick to render on ancient computers



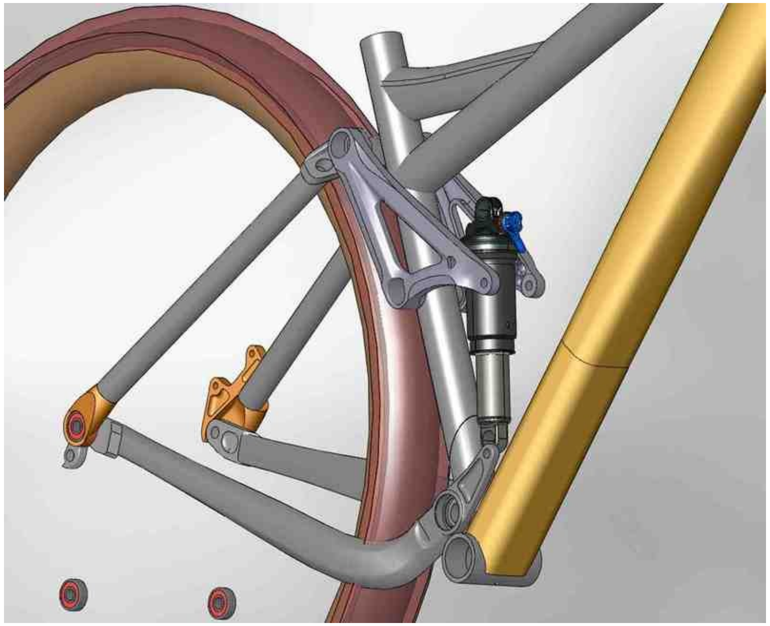
Neither of these matters today

But the Polya urn started out with more red balls than white...



Seeing what's there

There is one predominant technology – **the depth buffer.**



If everything had been F-Rep, not B-Rep, then the predominant technology would probably be ray-tracing.

Seeing what's there

The depth buffer.

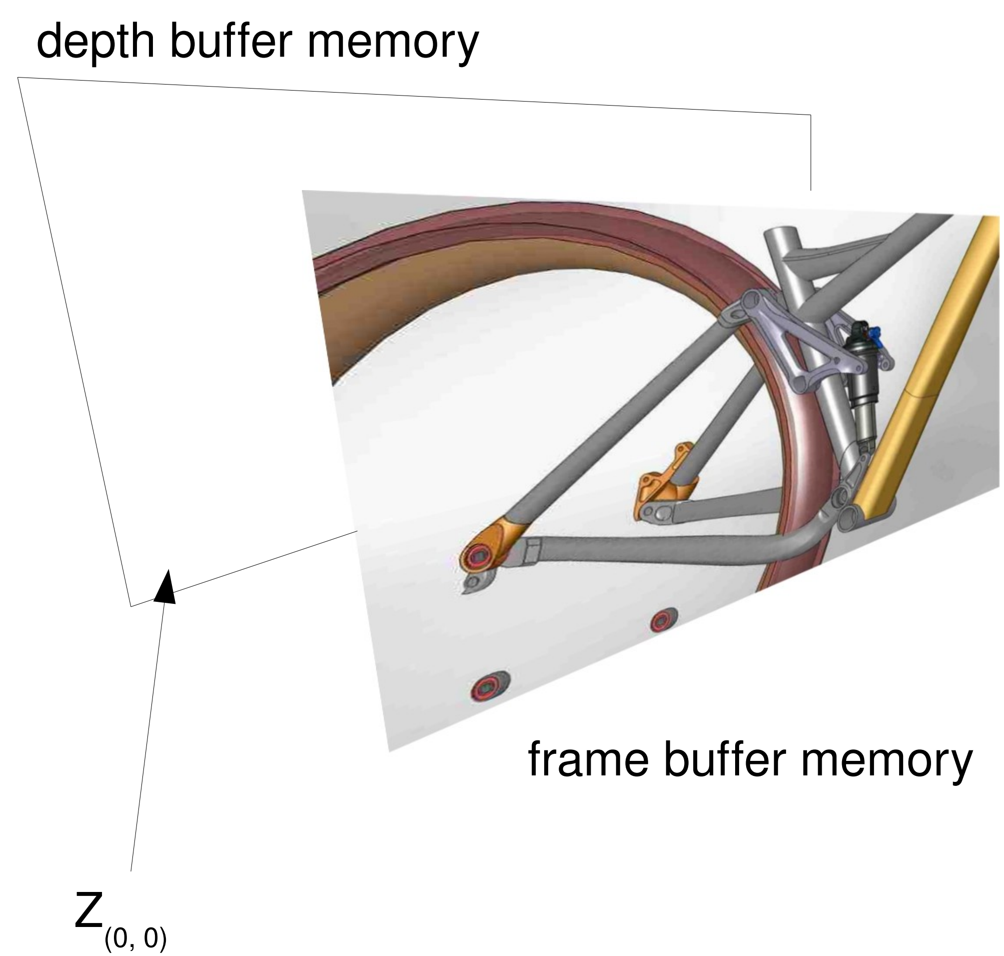
Entirely implemented in hardware courtesy of:

1. Silicon Graphics and flight simulators, then
2. Computer games

For each pixel store both colour and **depth**

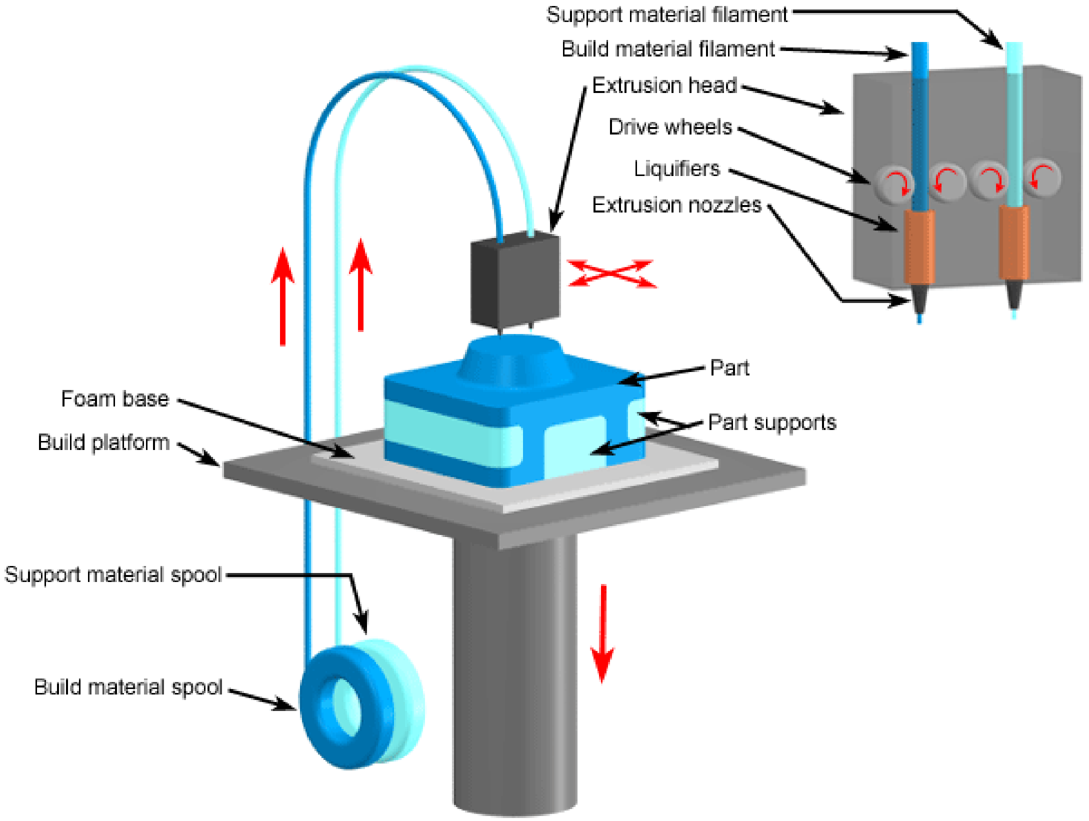
Send it **3D** coloured triangles in any order

Surfaces in front win



Computer Aided Manufacturing

Cutting away and building up



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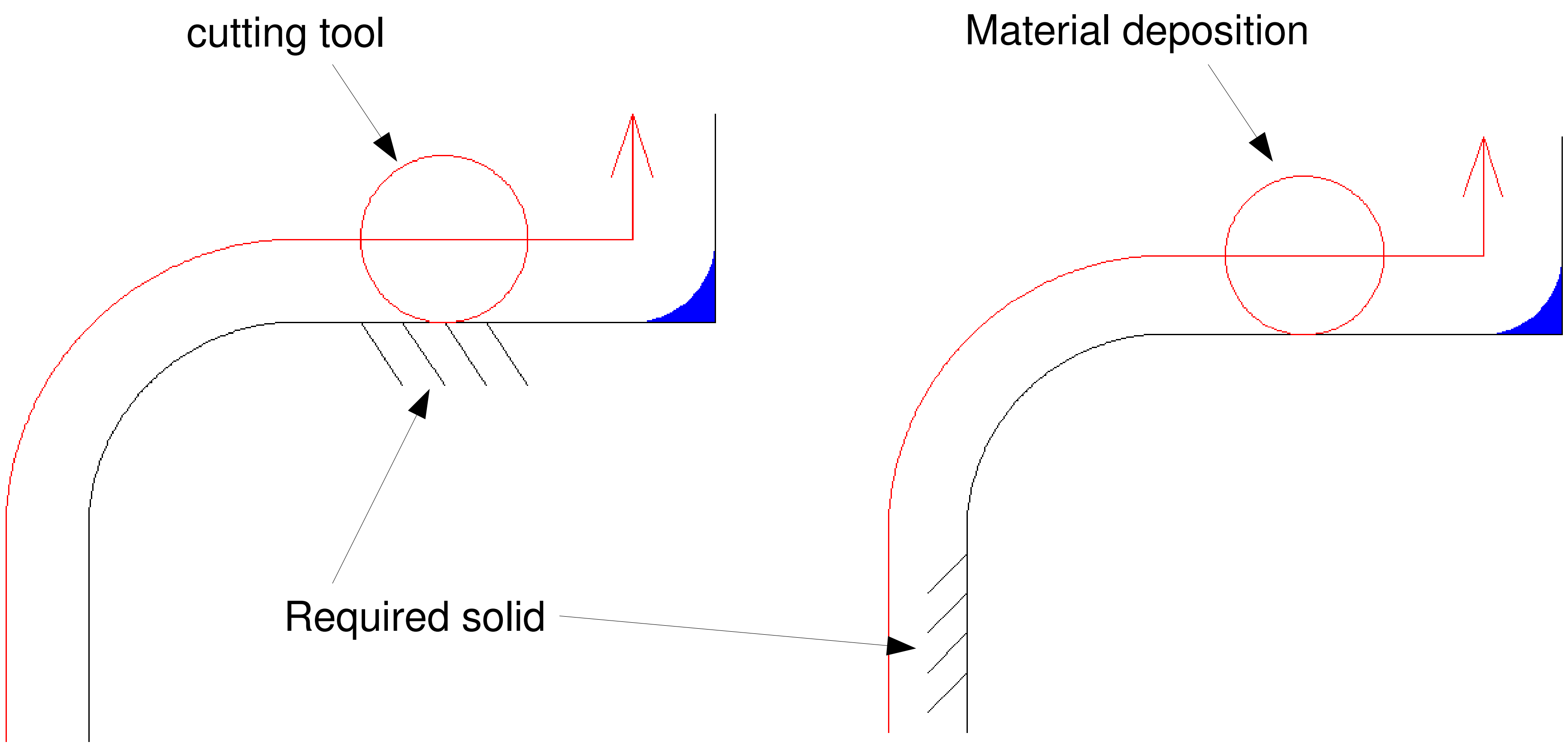
Cutting away and building up – both need **offsetting**



Offsetting straight lines and circular arcs is pretty straightforward.

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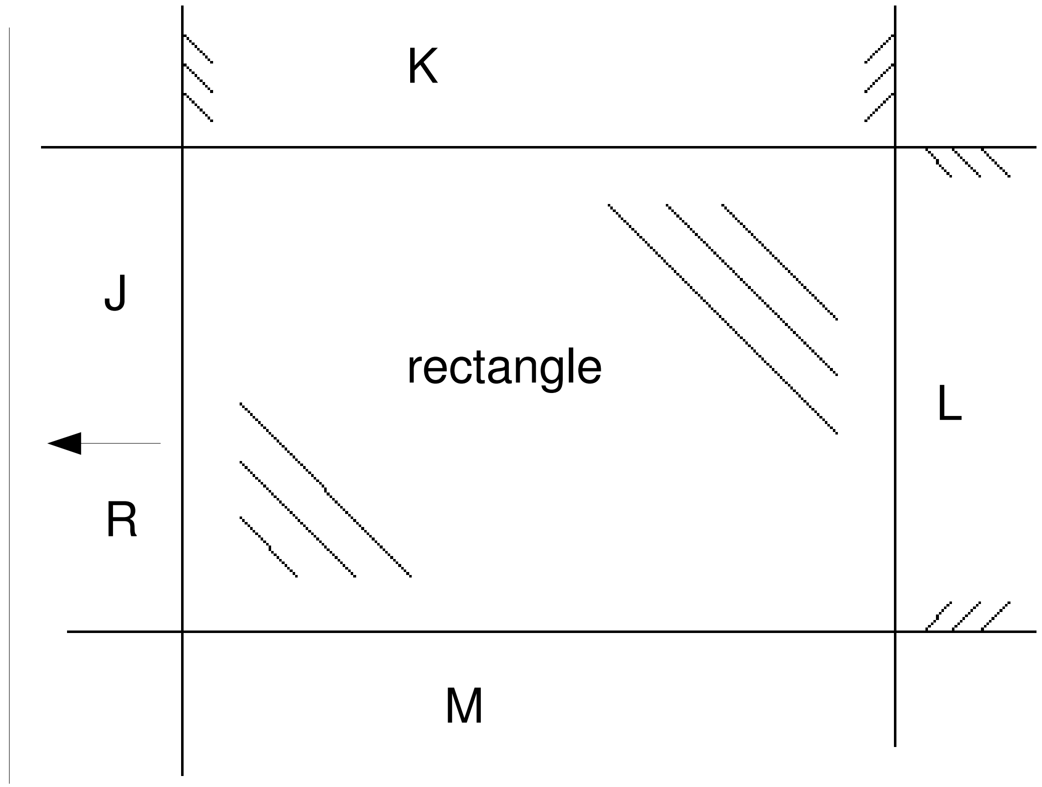
Offsetting **flat** F-Rep is very easy:



J: $Ax + By + C <= 0$ *etc.*

→

J': $Ax + By + C - R <= 0$ *etc.*

Offset rectangle: $J' \wedge K' \wedge L' \wedge M'$

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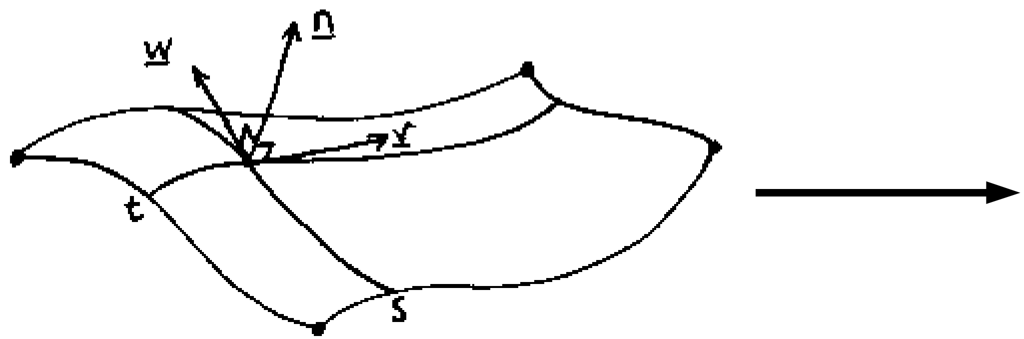
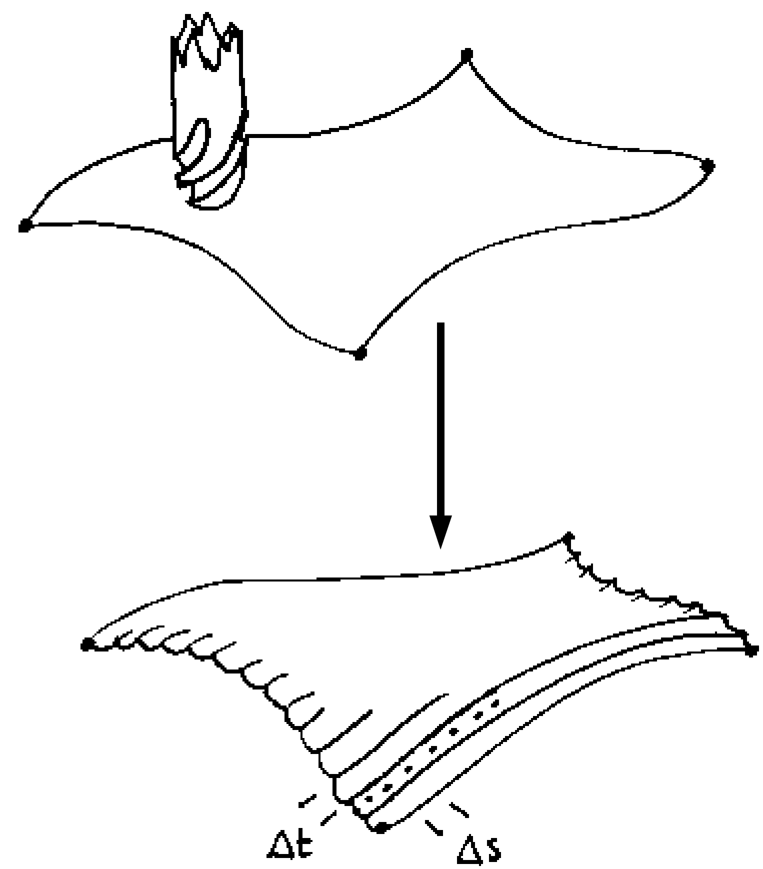
Offsetting parametric patches in B-Rep is surprisingly easy:



Ball-nosed tool, radius r



$\mathbf{P}(s, t) = (x(s, t), y(s, t), z(s, t))$

$\mathbf{W}(s, t) = \partial\mathbf{P}/\partial s, \quad \mathbf{V}(s, t) = \partial\mathbf{P}/\partial t$

$\mathbf{N}(s, t) = \mathbf{W} \times \mathbf{V}$

Tool centre is at $\mathbf{P} + r\,\mathbf{N}$ (**N** normalised).

Computer Aided Manufacturing

Offsetting pixel data is very easy:

Region growing

Set each white pixel within *r* of a black one red...

Very memory hungry in 3D

→

Quad/Oct-tree + interval tricks

Now it's not so easy as it was...

Computer Aided Manufacturing

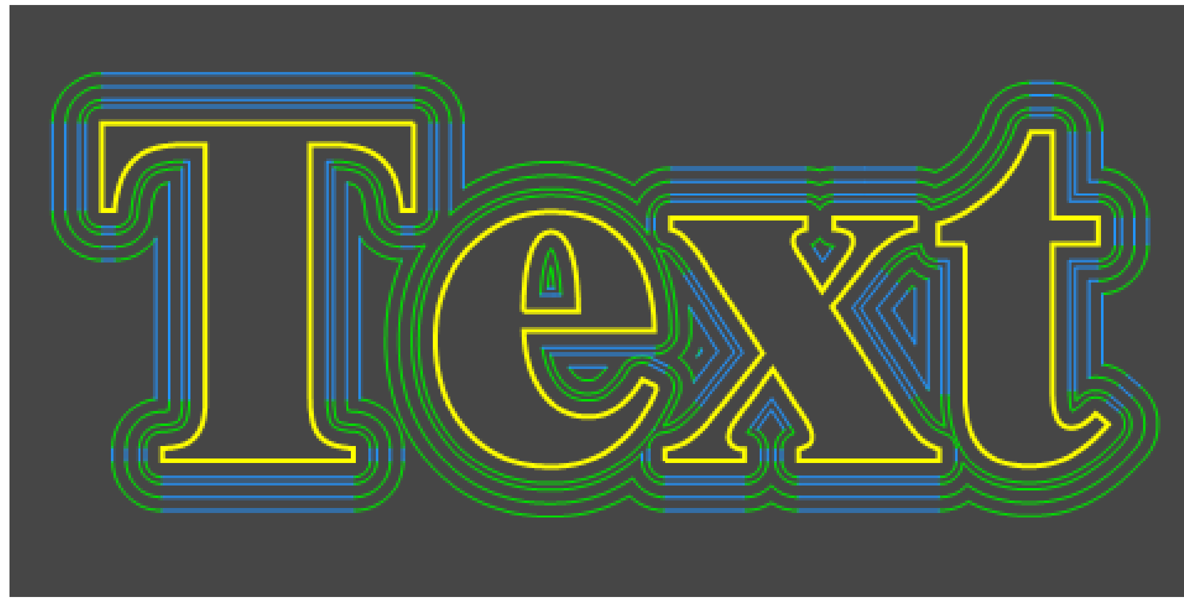
So. Offsetting is pretty much solved. What's the problem?

Toolpath collisions

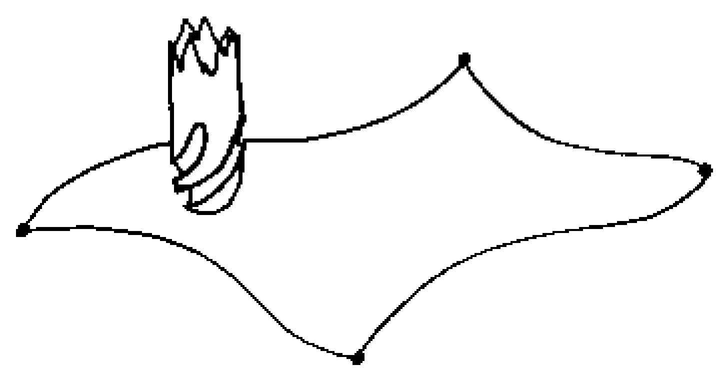
Not too hard to **detect**.

But what does the computer do about it?

Using a smaller tool sometimes works, but is inefficient.

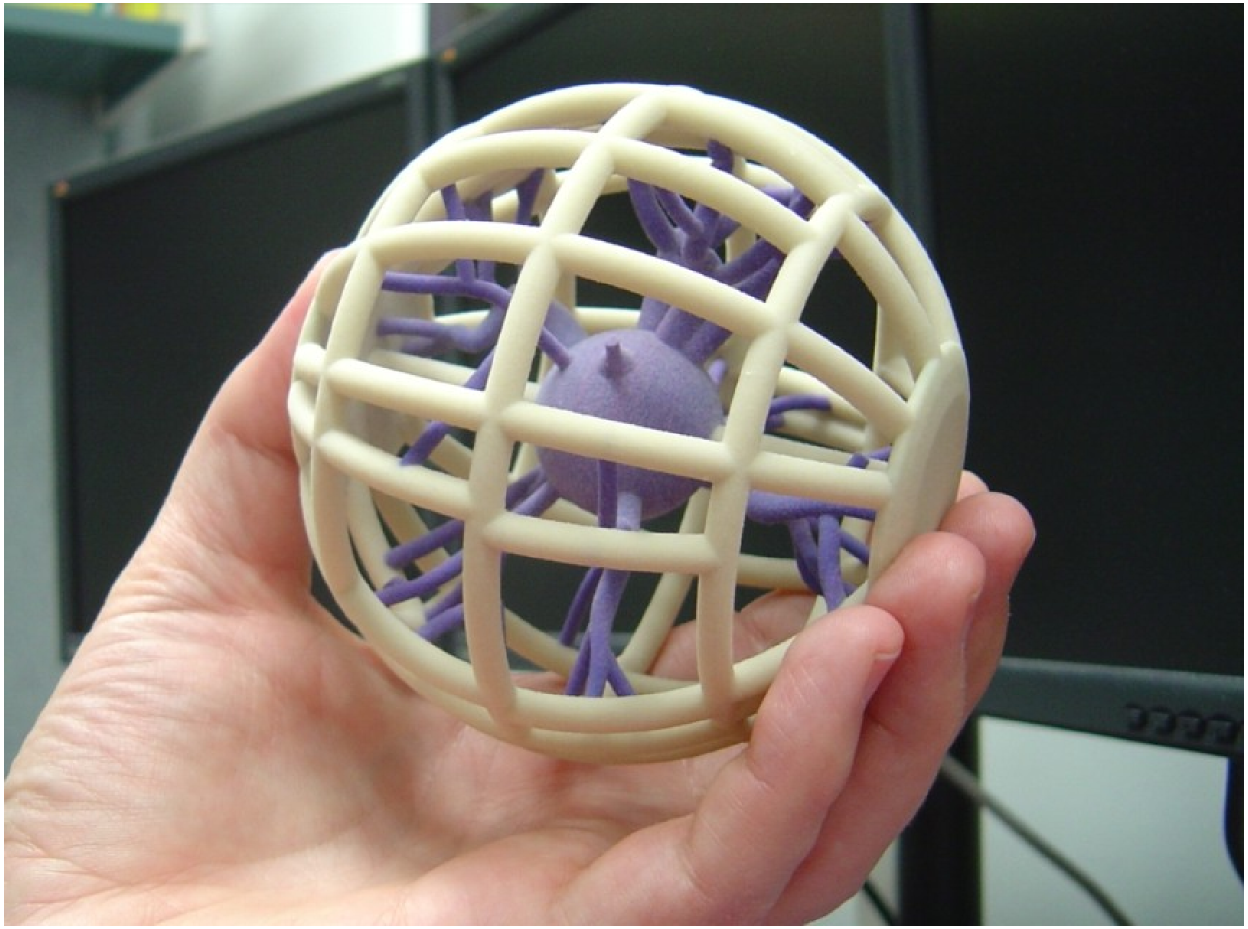


And what happens when the tool shaft hits the work?



Computer Aided Manufacturing

3D Toolpath collisions are not a problem at all for **additive manufacturing.**



This is the main reason that it is considered such a powerful technology.

Data exchange

How do we get data from CAD A to CAD B?

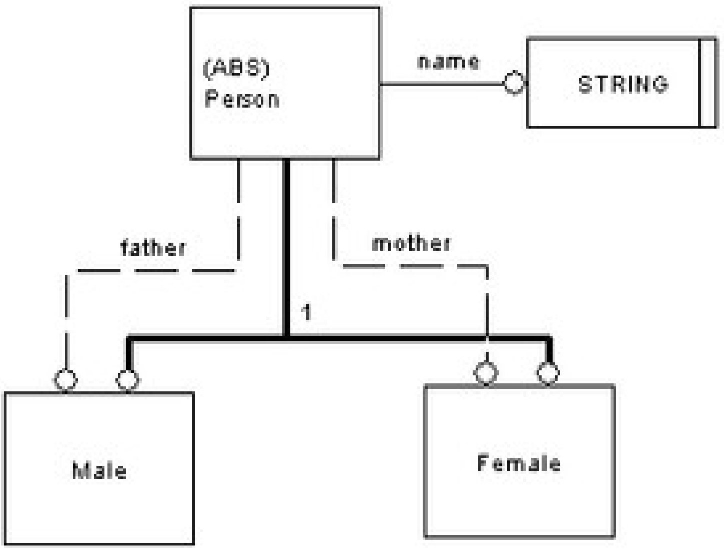
ISO 10303 - STEP

A definition language – Express

Implementation methods

Conformance tests

Etc.



Express-G

The biggest standard within ISO...

Data exchange

How do we get data from CAD to a numerically-controlled machine?

G-Codes

```
; GCode generated by RepRap Java Host Software
; Created: 2009-09-13:11-29-21
G21 ;metric
G90 ;absolute positioning
T0; select new extruder
G28; go home
M104 S190.0 ;set temperature
;#!LAYER: 1/48
M107 ;cooler off
G4 P20 ;delay
G1 Z0.0 F5.0 ;z move
G1 X1.7 Y2.2 F3000.0 ;horizontal move
```

Data exchange

How do we get data from CAD to additive-manufacturing machines?

STL files

```
solid
 facet normal n1 n2 n3
   outer loop
     vertex v11 v12 v13
     vertex v21 v22 v23
     vertex v31 v32 v33
   endloop
 endfacet
 facet normal 0.0 1.0 0.0
   outer loop
     Vertex 7.9 -2.63 12.2
     Vertex -18.4 -2.63 0.45
     Vertex 3.35 -2.63 5.5
   endloop
 endfacet
endsolid
```

In Conclusion

Neither CAD nor CAM is like logic or mathematics, but they are like the engineering they serve:

They don't always give the answer you want (nor even the correct answer) even if you put correct data in at the start. But they work almost all the time.

The predominant technologies are not necessarily the best nor the most elegant. They got here today by Polya-urn style historical accident.

References from the **Dawn of Time**...

Type most of the terms in this lecture into Google Scholar and you'll find all the modern stuff. But where did it all start?

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